Attachment 1.

THE QUARTERLY DCF MODEL, EFFECTIVE AND NOMINAL RATES OF RETURN AND THE DETERMINATION OF REVENUE REQUIREMENTS FOR REGULATED PUBLIC UTILITIES

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Authors in previous articles[1] have brought forth the concepts of effective and nominal rates of return and their relation to the determination of revenue requirements for regulated public utilities. It appears that much confusion exists concerning these concepts.[2] This paper will examine the appropriateness of the quarterly discounted cash flow (DCF) model and will expound upon the consistency of the various assumptions relating to: (1) the model, (2) adjusting the effective rate to the nominal rate, and (3) the proper determination of revenue requirements for regulated public utilities.

The DCF model is the most generally accepted method of measuring the cost of equity capital. According to DCF theory, the cost of equity is the discount rate (required rate) which equates the present value of the expected cash flows associated with a share of stock to the stock's price. The expected cash flows consist of expected dividends plus the price investors expect to receive when they sell the stock. The sales price in any period (t) will equal the present value of the dividends and sales price expected after period (t). Therefore, applying this concept to all future sales prices, the current stock price can be shown to equal the present value of all dividends expected to be paid in the future, including any liquidating dividend. This relationship can be expressed as:

$$P_0 = \frac{D_1}{(1+K)} + \frac{D_2}{(1+K)^2} + \frac{D_3}{(1+K)^3} + \ldots + \frac{D_\infty}{(1+K)^\infty} \qquad (1)$$

where: D_t = Dividend expected at the end of period t

K = Investor's required rate of return (the market cost of equity

P_0 = Price of stock at time zero

[1] For example see, C. M. Linke and J. K. Zumwalt, "Estimation Biases in Discounted Cash Flow Analyses of Equity Capital Cost in Rate Regulation," *Financial Management* (Autumn 1984): 15-20; J. J. Seigel, "The Application of the DCF Methodology for Determining the Cost of Equity Capital," *Financial Management* (Spring 1985): 46-51; and, C. M. Linke and J. K. Zumwalt, "The Irrelevance of Compounding Frequency in Determining a Utility's Cost of Equity," *Financial Management* (Autumn 1987): 65-69.

[2] See C. J. Cicchetti and J. D. Makholm, "The FERC's Discounted Cash Flow: A Compromise in the Wrong Direction," *Public Utilities Fortnightly*, July 9, 1987, 11-15; "Out of the Mailbag," *Public Utilities Fortnightly*, October 15, 1987 and May 16, 1988; and, "What Others Think," *Public Utilities Fortnightly*, February 4, 1988.

Assuming a constant growth in dividends (g) and g < k, equation (1) can be reduced to:

$$P_0 = \frac{D_1}{K - g} \tag{2}$$

which, after rearranging terms, results in the familiar infinite-horizon, constant-growth, annual DCF model:

$$K = \frac{D_1}{P_0} + g. \tag{3}$$

Equation (3) is predicated upon certain limiting assumptions including (1) annual dividend payment, (2) a constant rate of dividend growth, (3) P_0 determined on a dividend-payment date, and (4) an annual increase in dividends starting exactly one year in the future.

However, DCF models can be derived to evaluate cash flows of any periodicity (monthly, quarterly, annually, etc.) and/or growth. Any DCF model actually used should be derived to portray accurately the timing and amount of expected cash flows. Therefore, when dividends associated with common equity are paid on a quarterly basis, the investor's required return on common equity should be determined using a DCF model which reflects that quarterly payment.

A quarterly DCF model of the following form can be derived from equation (1):[3]

$$K = \frac{D_1(1+K)^{1-F_1} + D_2(1+K)^{1-F_2} + D_3(1+K)^{1-F_3} + D_4(1+K)^{1-F_4}}{P_0} + g \tag{4}$$

where: D_1 through D_4 = The quarterly dividends expected over the coming year.

F_1, F_2, F_3, F_4 = The fraction of the year before the quarterly dividends are received.

The quarterly DCF model recognizes that investors receive dividends quarterly and can reinvest them at their required rate of return. Therefore, when dividends are paid every three months, the quarterly model produces a higher required rate of return than the annual model, which assumes investors receive their dividends at the end of the year. For example, if the current price of a company's stock is $30.85, the

[3] Equation (5) assumes dividends will increase at a constant rate. If dividends are not expected to increase at a constant rate, a non-constant growth quarterly DCF model should be used. For an excellent analysis of constant growth and non-constant growth quarterly DCF models, including equation (5), see E. F. Brigham and T. C. Tapley, *Modifications to the DCF Stock Valuation Model*, (Public Utilities Research Center, Working Paper Series November 1984).

expected dividend during the coming year is \$2.80 (\$0.70 quarterly with the first payment one quarter away), the company's last dividend was paid at the old rate and dividends are expected to increase annually at a constant rate of 4.5 percent, the annual model produces a cost of equity of 13.58 percent:

$$K = \frac{D_1}{P_0} + G = \frac{\$\,2.80}{\$30.85} + 4.5\% = 13.58\%,$$

while the quarterly model produces a cost of equity of 14.04 percent:

$$K = \frac{.70(1+K)^{.75} + .70(1+K)^{.50} + .70(1+K)^{.25} + .70(1+K)^{0}}{\$30.85} + 4.5\% = 14.04\%$$

The 46-basis-point difference (14.04% - 13.58% = .46%) between the two models is due to the time value of money associated with investors receiving quarterly dividends, as recognized by the quarterly model, rather than annually, as assumed by the annual model. Depending on the circumstances, the annual model can significantly underestimate investors' required return.

Although theory indicates the quarterly model is the proper model to use in determining an investor's required rate of return for common equity when dividends are paid quarterly, it is the annual model that has been used most often in regulatory proceedings. However, use of the quarterly model is becoming more common. Consequently, regulators should recognize that the investor's required rate of return determined by the quarterly model (the effective rate) needs to be adjusted to a nominal rate for use in determining revenue requirements. The adjustment is needed because of the reinvestment assumptions associated with dividends paid and earnings retained.

The inapplicability of the use of an effective annual DCF required rate of return as the basis for computing utility revenue requirements can be shown best through an example using a certificate of deposit that is compounded several times a year. The nominal rate, although lower than the effective rate, is used to calculate the interest. The effective rate is used for comparison purposes and for calculating annual return requirements but cannot be applied--unadjusted--to the relevant principal balance.

<u>The Nominal Rate of Return and the Determination of Revenue Requirements</u>: The goal of regulation is to set rates so shareholders can expect to earn their required rate of return. As commonly practiced, a utility's required after-tax equity earnings are determined as the product of the

DCF cost of equity, the percentage of equity in the capital structure and the amount of rate base. However, as previously stated, the market-determined quarterly DCF cost of equity--the effective rate--needs adjustment to recognize the reinvestment assumptions associated with dividends paid and earnings retained. By making this adjustment, the determination of the test-period equity ratio used in calculating the revenue requirement will be consistent with the assumption inherent in the quarterly DCF cost-of-equity calculation regarding the utility's accumulation of equity and payment of dividends.

The adjustment is made by determining the n-period compounded equivalent (the nominal rate) of the quarterly DCF required rate of return. The number of compounding periods used in determining the nominal rate should equal the number of compounding periods used in compounding the utility's reinvestment of accumulated equity. The n-period compounded equivalent of the quarterly DCF required rate of return can be calculated using the following equation:

$$K_{rate} = [(1 + K_{qDCF})^{1/n} - 1] \times 12 \qquad (5)$$

where: K_{rate} = The rate making rate of return

K_{qDCF} = The quarterly DCF required rate of return

n = The number of compounding periods

By applying the nominal rate to each future, beginning-of-the-period equity balance, the utility will be provided with enough revenue to meet investor return requirements. However, if the effective required rate of return is applied to each future, beginning-of-the-period equity balance, the utility will be provided with more revenue than necessary to meet investor return requirements. This can be illustrated easily. Using the previous example, the quarterly model produced an effective required return on equity of 14.04 percent based on a stock price of $30.85, coming-year expected dividends of $2.80 and an expected growth rate of 4.5 percent. The assumptions inherent in this example are:

1. Stock price appreciation of $1.38825 ($30.85 x .045)
2. Earnings per share of $4.18825 ($1.38825 + $2.80)
3. Payout ratio of 66.85 percent ($2.80/$4.18825)
4. Growth in book value of 4.5 percent.

As shown in figure 1, applying the unadjusted quarterly DCF required rate of return to the beginning-of-the-period equity balance associated with each month of the test period produces a revenue requirement greater than necessary to meet the investor requirements inherent in the DCF cost of equity calculation.

As can be seen, the assumptions concerning earnings per share, the payout ratio, the growth in book value and the growth in per-share stock price are not consistent with those inherent in the DCF cost-of-equity calculation. This is due to the compounding associated with the utility's reinvestment of accumulated equity. Therefore, the DCF required rate of return--again, the effective rate--should be adjusted to a nominal rate of return to recognize the compounding associated with dividends paid and retained earnings. The sum of dividends, earnings on reinvested dividends and earnings on the utility's equity will result in the investor earning the required effective rate of return.

As shown in figure 2, applying the ratemaking rate of return of 13.21 percent,

$$13.21\% = [(1 + .1404)^{1/12} - 1] \times 12$$

to the beginning of the period equity balance associated with each month of the test period provides the utility with only the revenue necessary to meet the investor requirements inherent in the DCF cost of equity calculation.

The required rate of return used in determining revenue requirements may also need adjustment to accommodate the equity-ratio construct mandated by the regulatory agency (that is, 13-month-average, 12-month-average, etc.).

The ratemaking rate of return can be determined for any equity-ratio construct used by a regulatory agency simply by relating the required after-tax equity earnings as determined in figure 2, to the equity-ratio construct used by the regulatory agency. This relationship can be expressed as follows:

$$K_{rate} = \frac{\text{Required after-tax equity earnings}}{\text{Equity-ratio construct}} \qquad (6)$$

Continuing with the previous example and assuming a 13-month average rate base/equity ratio-construct, the ratemaking rate of return is 13.19 percent:

$$K_{rate} = \frac{\$\ 13,576}{\$102,898} = 13.19\%$$

Figure 3 shows that the use of the 13-month average rate base and the 13.9 percent ratemaking rate of return produce results consistent with the assumptions inherent in the DCF cost of equity calculation and those achieved in figure 2.

Inherent in equation (6) is the assumption that earnings will occur evenly over the year. This is a simplifying assumption that may not always be valid. For example, most natural gas distributors' earnings may occur during the winter months. Therefore, to account for an uneven earnings pattern, a weighted monthly equity factor should reflect the expected pattern of earnings. Figure 4 shows how the monthly equity factor can be determined using the expected monthly earnings to develop monthly weights. The weights are then used, as in equation (8), to solve through an iterative process for the nominal rate.

Figure 5 shows that application of the weighted monthly equity factor to the expected beginning-of-the-period equity balances will provide the investor with the effective required return on his initial investment. The effects of compounding and the size of the continuing equity balance will be affected by the degree to which earnings are slanted toward either the beginning or end of the test period. As shown in figure 5, when the earnings are concentrated in the beginning of the year, the adjustment to accommodate the 13-month average equity-ratio construct decreases the nominal rate by approximately 22 basis points.

Conclusions: The DCF model used to determine an investor's required rate of return should be derived to portray accurately the timing and amount of expected cash flows. When the dividends associated with common equity are paid on a quarterly basis, the investor's required return on common equity should be determined using a quarterly DCF model. However, the required rate of return determined by the DCF model--the effective rate--should be adjusted to a nominal rate of return for use in determining revenue requirements. The need for the adjustment is due to the reinvestment assumptions associated with dividends paid and earnings retained and certain regulatory practices regarding the equity ratio construct.

By applying the nominal rate to each future, beginning-of-the-period equity balance, the utility will be provided with only enough revenue to meet the investor return requirements inherent in the DCF cost of equity calculation. All other things being equal, ratepayers will not be overcharged or undercharged. Furthermore, given the amount of investment made by public utilities, proper determination of their allowed rate of return and the associated revenue requirements can have a significant effect.

MONTH (1)	COMMON EQUITY (2)	MONTHLY COST OF EQUITY FACTOR (3)	REV. REQ. (4)	EPS (5)	DPS (6)	P/O RATIO (7)	STOCK PRICE (8)
DEC 31,19X1	100000						30.85
JAN 31,19X2	101170	0.01170075	1170	0.3610			31.21
FEB 29	102354	0.01170075	1184	0.3652			31.58
MAR 31	101282	0.01170075	1198	0.3695	0.70	0.6389	31.25
APR 30	102467	0.01170075	1185	0.3656			31.61
MAY 31	103666	0.01170075	1199	0.3699			31.98
JUNE 30	102610	0.01170075	1213	0.3742	0.70	0.6308	31.66
JULY 31	103811	0.01170075	1201	0.3704			32.03
AUG 31	105026	0.01170075	1215	0.3747			32.40
SEPT 30	103985	0.01170075	1229	0.3791	0.70	0.6227	32.08
OCT 31	105202	0.01170075	1217	0.3754			32.45
NOV 30	106433	0.01170075	1231	0.3797			32.83
DEC 31	105409	0.01170075	1245	0.3842	0.70	0.6144	32.52
TOTAL			14486	4.4688	2.80	0.6266	

	DCF Analysis	Exhibit 1
DPS	$2.80	$2.80
EPS	$4.18825	$4.4688
P/O Ratio	66.85%	62.66%
Stock Price Appreciation	$1.38825	$1.6688
End of Period Book Value	$104,500	$105,409

Notes:
1) Monthly factor = Market required rate of return of 14.04% : 12 = .01170075
2) Previous month's ending balance is current month's beginning balance
3) Assumes 100% equity financing. For firms not 100% equity financed the amount of equity would be used to determine the equity ratio.
4) Monthly compounding of accumulated earnings.

Fig. 1. Company is allowed the quarterly DCF required rate of return on the beginning of the period equity balance associated with each month of the test period. For expository convenience the examples presented ignore flotation costs. For an excellent analysis regarding the need for a flotation cost adjustment see E. F. Brigham, D. A. Aberwald and L. C. Gapenski, "Common Equity Flotation Costs and Ratemaking," *Public Utilities Fortnightly*, May 2, 1985, 28-36.

MONTH (1)	COMMON EQUITY (2)	MONTHLY COST OF EQUITY FACTOR (3)	REV. REQ. (4)	EPS (5)	DPS (6)	P/O RATIO (7)	STOCK PRICE (8)
DEC 31,19X1	100000						30.85
JAN 31,19X2	101101	0.011009073	1101	0.3396			31.19
FEB 29	102214	0.011009073	1113	0.3434			31.53
MAR 31	101070	0.011009073	1125	0.3471	0.70	0.6795	31.18
APR 30	102183	0.011009073	1113	0.3433			31.52
MAY 31	103308	0.011009073	1125	0.3470			31.87
JUNE 30	102176	0.011009073	1137	0.3509	0.70	0.6723	31.52
JULY 31	103301	0.011009073	1125	0.3470			31.87
AUG 31	104438	0.011009073	1137	0.3508			32.22
SEPT 30	103319	0.011009073	1150	0.3547	0.70	0.6650	31.87
OCT 31	104456	0.011009073	1137	0.3509			32.22
NOV 30	105606	0.011009073	1150	0.3548			32.58
DEC 31	104500	0.011009073	1163	0.3587	0.70	0.6577	32.24
TOTAL			13576	4.1882	2.80	0.6685	

	DCF Analysis	Exhibit 2
DPS	$2.80	$2.80
EPS	$4.18825	$4.18825
P/O Ratio	66.85%	66.85%
Stock Price Appreciation	$1.38825	$1.38825
End of Period Book Value	$104,500	$104,500

Notes:
1) Monthly factor = Ratemaking rate of return of 13.21% : 12 = .011009073
2) Previous month's ending balance is current month's beginning balance
3) Assumes 100% equity financing. For firms not 100% equity financed the amount of equity would be used to determine the equity ratio.
4) Monthly compounding of accumulated earnings.

Fig. 2. Company is allowed the ratemaking rate of return on the beginning of the period equity balance associated with each month of the test period

MONTH (1)	COMMON EQUITY (2)	MONTHLY COST OF EQUITY FACTOR (3)	REV. REQ. (4)	EPS (5)	DPS (6)	P/O RATIO (7)	STOCK PRICE (8)
DEC 31,19X1	102898						30.85
JAN 31,19X2	102898	0.01099477	1131	0.3490			31.20
FEB 29	102898	0.01099477	1131	0.3490			31.55
MAR 31	102898	0.01099477	1131	0.3490	0.70	0.6685	31.20
APR 30	102898	0.01099477	1131	0.3490			31.55
MAY 31	102898	0.01099477	1131	0.3490			31.90
JUNE 30	102898	0.01099477	1131	0.3490	0.70	0.6685	31.54
JULY 31	102898	0.01099477	1131	0.3490			31.89
AUG 31	102898	0.01099477	1131	0.3490			32.24
SEPT 30	102898	0.01099477	1131	0.3490	0.70	0.6685	31.89
OCT 31	102898	0.01099477	1131	0.3490			32.24
NOV 30	102898	0.01099477	1131	0.3490			32.59
DEC 31	102898	0.01099477	1131	0.3490	0.70	0.6685	32.24
TOTAL			13576	4.1882	2.80	0.6685	

Notes: 1) Monthly factor = Ratemaking rate of return for 13-month average rate base/equity ratio of 13.19% : 12 = .01099477

2) Previous month's ending balance is current month's beginning balance

3) Assumes 100% equity financing. For firms not 100% equity financed the amount of equity would be used to determine the equity ratio.

4) Monthly compounding of accumulated earnings.

5) Mkt/Bk ratio calculated using actual book value and not allowed average book value.

Fig. 3. Company is allowed the ratemaking rate of return on the 13-month average equity rate base

		AS A PERCENT OF TOTAL	
MONTH	EARNINGS	EARNINGS	CUMULATIVE PERCENT
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JANUARY	$2,245	15.99%	15.99%
FEBRUARY	$2,000	14.25%	30.24%
MARCH	$1,695	12.07%	42.31%
APRIL	$900	6.41%	48.72%
MAY	$900	6.41%	55.13%
JUNE	$900	6.41%	61.54%
JULY	$900	6.41%	67.95%
AUGUST	$900	6.41%	74.36%
SEPTEMBER	$900	6.41%	80.77%
OCTOBER	$900	6.41%	87.18%
NOVEMBER	$900	6.41%	93.59%
DECEMBER	$900	6.41%	100.00%
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	$14,040	100.00%	

INVESTOR'S REQUIRED RETURN = 14.04%

INVESTOR'S REQUIRED RETURN = ((1+(MW1xNR))x(1+(MW2xNR))x(1+(MW3xNR))x (8)
(1+(MW4xNR))x(1+(MW5xNR))x(1+(MW6xNR))x
(1+(MW7xNR))x(1+(MW8xNR))x(1+(MW9xNR))x
(1+(MW10xNR))x(1+(MW11xNR))x(1+(MW12xNR))) - 1

WHERE:

MW = MONTHLY WEIGHT(MONTHLY EARNINGS AS A PERCENT OF TOTAL EARNINGS)
NR = NOMINAL RATE OF RETURN(SOLVED ITERATIVELY)

14.04% = ((1+(.1599 x .132222226))x(1+(.1425 x .132222226))x(1+(.1207 x .132222226))x
(1+(.0641 x .132222226))x(1+(.0641 x .132222226))x(1+(.0641 x .132222226))x
(1+(.0641 x .132222226))x(1+(.0641 x .132222226))x(1+(.0641 x .132222226))x
(1+(.0641 x .132222226))x(1+(.0641 x .132222226))x(1+(.0641 x .132222226))) - 1

Fig. 4. Determination of nominal rate of return using expected monthly revenue flows to develop weights

EXHIBIT 5: REVENUES CONCENTRATED AT THE BEGINNING OF THE YEAR

		YEAR 1 COMMON EQUITY	MONTHLY EQUITY FACTOR	DOLLAR EARNINGS
BEGINNING BALANCE		$100,000.00		
MONTH	JANUARY	$102,114.23	0.0211423339	$2,114.23
	FEBRUARY	$104,038.24	0.0188416672	$1,924.00
	MARCH	$105,698.61	0.0159592227	$1,660.37
	APRIL	$106,594.45	0.0084754447	$895.84
	MAY	$107,497.88	0.0084754447	$903.44
	JUNE	$108,408.98	0.0084754447	$911.09
	JULY	$109,327.79	0.0084754447	$918.81
	AUGUST	$110,254.39	0.0084754447	$926.60
	SEPTEMBER	$111,188.85	0.0084754447	$934.45
	OCTOBER	$112,131.22	0.0084754447	$942.37
	NOVEMBER	$113,081.58	0.0084754447	$950.36
	DECEMBER	$114,040.00	0.0084754447	$958.42
TOTAL			0.132222226	$14,040.00
13-MONTH AVERAGE		$108,028.94		

REQUIRED RETURN ON EQUITY (EFFECTIVE RATE)		= 14.04%
REQUIRED DOLLAR RETURN ON INITIAL INVESTMENT	= 14.04% X $100,000	= $14,040
APPLICATION OF RATEMAKING RATE OF RETURN (WEIGHTED BY EXPECTED EARNINGS FLOW)		= $14,040
RATEMAKING RATE OF RETURN ADJUSTED FOR 13-MONTH AVERAGE RATEBASE	= $14,040/$108,028.94	= 12.9965174%
APPLICATION OF ADJUSTED RATEMAKING RATE OF RETURN TO 13-MONTH AVERAGE BALANCE	= 12.9965174% X $108,937.57	= $14,040

Fig. 5. Revenues concentrated at the beginning of the year